

CHAPTER 6: SYSTEMS OF PARTICLES AND ROTATIONAL MOTION

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6.1 INTRODUCTION

In earlier chapters, we primarily considered the motion of a single particle (ideally represented as a point mass having no size). We applied the results even to bodies of finite size, assuming their motion could be described in terms of particle motion.

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Extended Bodies and Systems of Particles

Any real body which we encounter in daily life has a **finite size**. In dealing with the motion of extended bodies, the idealized model of a particle is often inadequate.

Key Concepts:

- ✓ An extended body is a **system of particles**
- ✓ **Centre of mass** - a key concept for understanding motion
- ✓ Motion of centre of mass represents translational motion of the system
- ✓ Motion about centre of mass represents rotational motion

Rigid Bodies

RIGID BODY: A body with a perfectly definite and unchanging shape

The distances between all pairs of particles do not change

Real vs Ideal Rigid Bodies

Important Note: No real body is truly rigid - all real bodies deform under the influence of forces.

When can we treat a body as rigid?

- When deformations are negligible
- Examples: wheels, tops, steel beams, molecules, planets
- We ignore warping, bending, or vibrations

Types of Motion for Rigid Bodies

◆ Pure Translation

At any instant of time, **all particles of the body have the same velocity**

Example: A block sliding down an inclined plane without sidewise movement

◆ Pure Rotation (about a fixed axis)

Every particle moves in a circle which:

- Lies in a plane perpendicular to the axis
- Has its centre on the axis
- All particles have the same angular velocity

Examples: Ceiling fan, potter's wheel, merry-go-round

◆ Combined Motion

Combination of translation and rotation

Example: A cylinder rolling down an inclined plane

- Translational component - movement down the plane
- Rotational component - spinning about its axis
- Different particles have different velocities at any instant

6.2 CENTRE OF MASS

The centre of mass of a system of particles is that point which moves as though all the mass of the system were concentrated at that point and all external forces were applied at that point.

Two-Particle System

Consider two particles of masses m_1 and m_2 at positions x_1 and x_2 from origin O.

Centre of mass position X:

$$X = (m_1x_1 + m_2x_2)/(m_1 + m_2)$$

X is the **mass-weighted mean** of x_1 and x_2

Special Case: If $m_1 = m_2 = m$ (equal masses)

$$X = (x_1 + x_2)/2$$

Centre of mass lies exactly midway between them

System of n Particles (Along a Line)

$$X = (m_1x_1 + m_2x_2 + \dots + m_nx_n)/(m_1 + m_2 + \dots + m_n)$$

$$X = (\sum m_i x_i)/(\sum m_i) = (\sum m_i x_i)/M$$

where M = total mass of system

Three-Dimensional System

For particles not lying in a straight line, we need x, y, and z coordinates:

$$\mathbf{X} = (\Sigma m_i x_i)/M$$

$$\mathbf{Y} = (\Sigma m_i y_i)/M$$

$$\mathbf{Z} = (\Sigma m_i z_i)/M$$

Vector Form:

Position vector of i^{th} particle: $\mathbf{r}_i = x_i \hat{i} + y_i \hat{j} + z_i \hat{k}$

Position vector of CM: $\mathbf{R} = X \hat{i} + Y \hat{j} + Z \hat{k}$

$$\mathbf{R} = (\Sigma m_i \mathbf{r}_i)/M$$

For Equal Masses

If three particles have equal mass m at vertices of a triangle:

- $X = (x_1 + x_2 + x_3)/3$
- $Y = (y_1 + y_2 + y_3)/3$
- **Centre of mass coincides with the centroid of the triangle**

Continuous Mass Distribution

For rigid bodies (large number of particles, closely packed):

Replace summations with integrals:

$$X = (1/M) \int x \, dm$$

$$Y = (1/M) \int y \, dm$$

$$Z = (1/M) \int z \, dm$$

$$\mathbf{R} = (1/M) \int \mathbf{r} \, dm$$

where $M = \int dm$ is the total mass

Symmetry and Centre of Mass

For homogeneous bodies with regular shapes:

- ✓ Centre of mass coincides with **geometric centre**
- ✓ This follows from reflection symmetry
- ✓ Examples: uniform rods, rings, discs, spheres, cubes

Why? For every element dm at position r , there's an element of same mass at position $-r$, making $\int r \, dm = 0$

Example 6.1: Centre of Mass of Equilateral Triangle

Question: Find the centre of mass of three particles at the vertices of an equilateral triangle. Masses are 100g, 150g, and 200g. Each side = 0.5m.

Solution:

Set up coordinates: O at origin, A at (0.5, 0), B at (0.25, $0.25\sqrt{3}$)

X-coordinate:

$$X = [100(0) + 150(0.5) + 200(0.25)] / (100 + 150 + 200)$$

$$= (75 + 50) / 450 = 125 / 450 = \mathbf{5/18 \text{ m}}$$

Y-coordinate:

$$Y = [100(0) + 150(0) + 200(0.25\sqrt{3})] / 450$$

$$= 50\sqrt{3} / 450 = \mathbf{\sqrt{3}/9 \text{ m}}$$

Note: Centre of mass is NOT at the geometric centre because masses are unequal

Example 6.2: Centre of Mass of Triangular Lamina

Method: Subdivide into narrow strips parallel to base

Key Observation:

- By symmetry, each strip has CM at its midpoint
- Joining all midpoints gives the median
- CM must lie on all three medians
- **Therefore, CM is at the centroid (intersection of medians)**

Example 6.3: Centre of Mass of L-Shaped Lamina

Question: Find CM of uniform L-shaped lamina (mass 3 kg) with dimensions as shown

Solution:

Divide into 3 squares (1m × 1m each), mass of each = 1 kg

CM of each square (by symmetry):

- $C_1: (1/2, 1/2)$
- $C_2: (3/2, 1/2)$
- $C_3: (1/2, 3/2)$

Overall CM:

$$X = [1(1/2) + 1(3/2) + 1(1/2)]/3 = (1/2 + 3/2 + 1/2)/3 = \mathbf{5/6 \text{ m}}$$

$$Y = [1(1/2) + 1(1/2) + 1(3/2)]/3 = (1/2 + 1/2 + 3/2)/3 = \mathbf{5/6 \text{ m}}$$

Note: CM lies on the diagonal due to symmetry about diagonal



6.3 MOTION OF CENTRE OF MASS

We now explore the physical importance of the centre of mass for a system of particles.

Starting from position vector:

$$MR = m_1r_1 + m_2r_2 + \dots + m_nr_n = \sum m_i r_i$$

Differentiating with respect to time:

$$M(dR/dt) = \sum m_i (dr_i/dt)$$

$$\mathbf{MV} = m_1\mathbf{v}_1 + m_2\mathbf{v}_2 + \dots + m_n\mathbf{v}_n = \sum m_i\mathbf{v}_i$$

where $V = dR/dt$ is velocity of centre of mass

Differentiating again:

$$M(dV/dt) = \sum m_i (dv_i/dt)$$

$$\mathbf{MA} = m_1\mathbf{a}_1 + m_2\mathbf{a}_2 + \dots + m_n\mathbf{a}_n = \sum m_i\mathbf{a}_i$$

where $A = dV/dt$ is acceleration of centre of mass

From Newton's Second Law:

Force on i^{th} particle: $F_i = m_i a_i$

Therefore:

$$MA = F_1 + F_2 + \dots + F_n = \Sigma F_i$$

Forces include external AND internal forces.

Key Result: Internal forces occur in equal and opposite pairs (Newton's Third Law), so their sum is zero.

$$\mathbf{MA} = \mathbf{F}_{\text{ext}}$$

where F_{ext} = sum of all external forces

 **Newton's Second Law for System of Particles**

The centre of mass of a system moves as if:

- ✓ All the mass of the system was concentrated at the CM
- ✓ All external forces were applied at that point

Important: No knowledge of internal forces is required to determine CM motion!

Projectile Explosion Example

A projectile follows a parabolic trajectory. It explodes mid-air into fragments.

What happens to the CM?

- Explosion forces are internal
- Only external force is gravity (same before and after)
- **CM continues along the same parabolic path!**

6.4 LINEAR MOMENTUM OF A SYSTEM OF PARTICLES

Linear momentum of i^{th} particle: $p_i = m_i v_i$

Total linear momentum:

$$P = p_1 + p_2 + \dots + p_n = \Sigma m_i v_i$$

From $MV = \sum m_i v_i$, we get:

$$\mathbf{P} = M\mathbf{V}$$

Total momentum = (Total mass) \times (Velocity of CM)

Differentiating:

$$dP/dt = M(dV/dt) = MA$$

Since $MA = F_{\text{ext}}$:

$$d\mathbf{P}/dt = \mathbf{F}_{\text{ext}}$$

This is Newton's Second Law extended to a system of particles

Conservation of Linear Momentum

If $\mathbf{F}_{\text{ext}} = \mathbf{0}$, then:

$$d\mathbf{P}/dt = 0$$

$\mathbf{P} = \text{constant}$

Law of Conservation of Linear Momentum

When the total external force on a system of particles is zero, the total linear momentum of the system is conserved.

This also means: When $F_{\text{ext}} = 0$, velocity of centre of mass remains constant

Component form:

- $P_x = \text{constant}$
- $P_y = \text{constant}$
- $P_z = \text{constant}$

Application: Radioactive Decay

Radium nucleus \rightarrow Radon nucleus + Alpha particle

Analysis:

- Decay forces are internal
- External forces negligible
- Total momentum conserved
- **CM continues with same velocity**

In CM frame: Product particles move back-to-back with CM at rest



6.5 VECTOR PRODUCT OF TWO VECTORS

Vector product (also called cross product) is essential for defining torque and angular momentum.

$$\text{VECTOR PRODUCT: } \mathbf{c} = \mathbf{a} \times \mathbf{b}$$

$$\text{Magnitude: } c = ab \sin\theta$$

Direction: Given by right-hand screw rule

Properties of Vector Product

◆ Direction (Right-Hand Screw Rule)

Method 1: Turn a right-handed screw from \mathbf{a} to \mathbf{b} through smaller angle. The direction of advancement gives direction of $\mathbf{a} \times \mathbf{b}$

Method 2 (Right-Hand Rule): Curl fingers from \mathbf{a} to \mathbf{b} . Stretched thumb points in direction of $\mathbf{a} \times \mathbf{b}$

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💡 Key Properties

1. NOT Commutative:

$$a \times b \neq b \times a$$

Instead: $a \times b = -(b \times a)$

2. Distributive:

$$a \times (b + c) = a \times b + a \times c$$

3. Behavior Under Reflection:

Under reflection: $a \rightarrow -a, b \rightarrow -b$

But: $a \times b \rightarrow (-a) \times (-b) = a \times b$

Vector product does not change sign under reflection!

Elementary Cross Products

$$\mathbf{a} \times \mathbf{a} = \mathbf{0} \text{ (null vector)}$$

Unit vectors:

$$\hat{i} \times \hat{i} = \hat{j} \times \hat{j} = \hat{k} \times \hat{k} = \mathbf{0}$$

$$\hat{i} \times \hat{j} = \hat{k}$$

$$\hat{j} \times \hat{k} = \hat{i}$$

$$\hat{k} \times \hat{i} = \hat{j}$$

Reverse order (anticommutative):

$$\hat{j} \times \hat{i} = -\hat{k}$$

$$\hat{k} \times \hat{j} = -\hat{i}$$

$$\hat{i} \times \hat{k} = -\hat{j}$$

Cyclic Rule

If $\hat{i}, \hat{j}, \hat{k}$ occur **cyclically**: product is positive

If they do NOT occur cyclically: product is negative

Component Form

Given:

$$a = a_x\hat{i} + a_y\hat{j} + a_z\hat{k}$$

$$b = b_x\hat{i} + b_y\hat{j} + b_z\hat{k}$$

Vector product:

$$a \times b = (a_y b_z - a_z b_y)\hat{i} + (a_z b_x - a_x b_z)\hat{j} + (a_x b_y - a_y b_x)\hat{k}$$

Determinant Form (easy to remember):

$$a \times b = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ a_x & a_y & a_z \\ b_x & b_y & b_z \end{vmatrix}$$

Example 6.4: Scalar and Vector Products

Given: $a = 3\hat{i} - 4\hat{j} + 5\hat{k}$ and $b = -2\hat{i} + \hat{j} - 3\hat{k}$

Scalar Product:

$$a \cdot b = (3)(-2) + (-4)(1) + (5)(-3)$$

$$= -6 - 4 - 15 = -25$$

Vector Product:

$$a \times b = |\hat{i} \hat{j} \hat{k}|$$

$$|3 \ -4 \ 5|$$

$$|-2 \ 1 \ -3|$$

$$= \hat{i}[(-4)(-3) - (5)(1)] - \hat{j}[(3)(-3) - (5)(-2)] + \hat{k}[(3)(1) - (-4)(-2)]$$

$$= \hat{i}(12 - 5) - \hat{j}(-9 + 10) + \hat{k}(3 - 8)$$

$$= 7\hat{i} - \hat{j} - 5\hat{k}$$

Verify: $b \times a = -7\hat{i} + \hat{j} + 5\hat{k} = -(a \times b) \checkmark$

6.6 ANGULAR VELOCITY AND LINEAR VELOCITY

Every particle of a rotating rigid body moves in a circle. The linear velocity is related to angular velocity through a vector product.

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☀ Rotation About Fixed Axis

Consider a rigid body rotating about a fixed axis (say z-axis):

- Each particle moves in a circle
- Circle lies in plane perpendicular to axis
- Centre of circle is on the axis
- All particles have same angular velocity ω

For particle at perpendicular distance r from axis:

Linear velocity magnitude: $v = \omega r$

This applies to ALL particles at any instant

🎯 Angular Velocity as a Vector

Angular velocity ω is a vector:

- Magnitude: $\omega = d\theta/dt$
- Direction: Along axis of rotation
- Sense: Right-hand screw rule (advance direction when rotating)

Relation Between v and ω

Key Result:

$$\mathbf{v} = \boldsymbol{\omega} \times \mathbf{r}$$

where r is position vector of particle from a point on the axis

Verification of magnitude:

- $|\boldsymbol{\omega} \times \mathbf{r}| = \omega r \sin\theta$
- $r \sin\theta = r_{\perp}$ (perpendicular distance from axis)
- Therefore: $|v| = \omega r_{\perp} \checkmark$

Direction: $\boldsymbol{\omega} \times \mathbf{r}$ is perpendicular to both $\boldsymbol{\omega}$ and \mathbf{r} , hence tangent to circle \checkmark

General Rotation

The relation $v = \boldsymbol{\omega} \times \mathbf{r}$ holds even for more general rotation with one point fixed (like a spinning top)

In this case, \mathbf{r} is position vector from the fixed point

Angular Acceleration

Angular Acceleration:

$$\alpha = d\omega/dt$$

For fixed axis: $\alpha = d\omega/dt$ (scalar form)

Example 6.6: Angular Momentum Conservation

Question: Show that angular momentum about any point of a single particle moving with constant velocity remains constant.

Solution:

Angular momentum: $l = r \times mv$

Magnitude: $l = mvr \sin\theta = mv(r \sin\theta)$

Key Observation: $r \sin\theta =$ perpendicular distance from O to line of motion = constant

Direction: Perpendicular to plane of r and v (into page) = constant

Conclusion: Both magnitude and direction constant, hence **l is conserved**

Note: No external torque acts on the particle

6.7 TORQUE AND ANGULAR MOMENTUM

Two fundamental vector quantities in rotational motion, defined as vector products.

Torque (Moment of Force)

$$\text{TORQUE: } \tau = \mathbf{r} \times \mathbf{F}$$

$$\text{Magnitude: } \tau = rF \sin\theta$$

Direction: Perpendicular to plane of \mathbf{r} and \mathbf{F} (right-hand rule)

Alternative expressions for magnitude:

$$\tau = rF \sin\theta = (r \sin\theta)F = r_{\perp}F$$

where r_{\perp} = perpendicular distance from origin to line of action of \mathbf{F}

$$\text{Also: } \tau = r(F \sin\theta) = rF_{\perp}$$

where F_{\perp} = component of \mathbf{F} perpendicular to \mathbf{r}

When is Torque Zero?

$\tau = 0$ if:

- $r = 0$ (force acts at origin)
- $F = 0$ (no force)
- $\theta = 0^{\circ}$ or 180° (force parallel/antiparallel to \mathbf{r})
- Equivalently: line of action passes through origin

💡 Dimensions and Units

Dimensions: $[M L^2 T^{-2}]$ (same as energy, but different quantity!)

SI Unit: $N \cdot m$ (newton metre)

Important: Torque is a vector; work is a scalar

🎯 Angular Momentum

ANGULAR MOMENTUM: $l = r \times p$

where $p = mv$ is linear momentum

Magnitude: $l = rp \sin\theta = mvr \sin\theta$

Alternative expressions:

$l = r_{\perp}p$ where $r_{\perp} = r \sin\theta$ (perpendicular distance from origin to line of momentum)

$l = rp_{\perp}$ where $p_{\perp} = p \sin\theta$ (component of p perpendicular to r)

Relation Between Torque and Angular Momentum

Starting from $\mathbf{l} = \mathbf{r} \times \mathbf{p}$:

Differentiate: $d\mathbf{l}/dt = d/dt(\mathbf{r} \times \mathbf{p})$

Using product rule: $d\mathbf{l}/dt = (d\mathbf{r}/dt) \times \mathbf{p} + \mathbf{r} \times (d\mathbf{p}/dt)$

Analysis:

- $d\mathbf{r}/dt = \mathbf{v}$, and $\mathbf{p} = m\mathbf{v}$
- Therefore: $\mathbf{v} \times (m\mathbf{v}) = m(\mathbf{v} \times \mathbf{v}) = 0$
- Also: $d\mathbf{p}/dt = \mathbf{F}$

$$d\mathbf{l}/dt = \mathbf{r} \times \mathbf{F} = \boldsymbol{\tau}$$

Time rate of change of angular momentum = Torque

This is the rotational analogue of $\mathbf{F} = d\mathbf{p}/dt$

System of Particles

Total angular momentum:

$$\mathbf{L} = \mathbf{l}_1 + \mathbf{l}_2 + \dots + \mathbf{l}_n = \Sigma(\mathbf{r}_i \times \mathbf{p}_i)$$

Total torque:

$$\boldsymbol{\tau} = \boldsymbol{\tau}_1 + \boldsymbol{\tau}_2 + \dots + \boldsymbol{\tau}_n = \Sigma(\mathbf{r}_i \times \mathbf{F}_i)$$

Key Result:

$$dL/dt = \Sigma(dl_i/dt) = \Sigma(r_i \times F_i)$$

Forces include external AND internal forces

Internal forces: Equal and opposite (Newton's Third Law), and act along line joining particles

Therefore: $\tau_{\text{int}} = 0$

$$dL/dt = \tau_{\text{ext}}$$

Time rate of total angular momentum = Total external torque

 **Conservation of Angular Momentum**

If $\tau_{\text{ext}} = 0$:

$$dL/dt = 0$$

L = constant

☀ Law of Conservation of Angular Momentum

If the total external torque on a system is zero, the total angular momentum is conserved.

Component form:

- $L_x = \text{constant}$
- $L_y = \text{constant}$
- $L_z = \text{constant}$

Example 6.5: Calculating Torque

Given: $F = 7\hat{i} + 3\hat{j} - 5\hat{k}$, $r = \hat{i} - \hat{j} + \hat{k}$

Find torque about origin:

$$\tau = r \times F = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \end{vmatrix}$$

$$\begin{vmatrix} 1 & -1 & 1 \end{vmatrix}$$

$$\begin{vmatrix} 7 & 3 & -5 \end{vmatrix}$$

$$= \hat{i}[(-1)(-5) - (1)(3)] - \hat{j}[(1)(-5) - (1)(7)] + \hat{k}[(1)(3) - (-1)(7)]$$

$$= \hat{i}(5 - 3) - \hat{j}(-5 - 7) + \hat{k}(3 + 7)$$

$$= 2\hat{i} + 12\hat{j} + 10\hat{k}$$



6.8 EQUILIBRIUM OF A RIGID BODY

A rigid body is in mechanical equilibrium when both its linear and angular momentum are not changing with time.

CONDITIONS FOR MECHANICAL EQUILIBRIUM:

1. Translational Equilibrium:

$$\Sigma F = 0$$

2. Rotational Equilibrium:

$$\Sigma T = 0$$

Scalar Equations

Six independent conditions (3D):

Translational:

- $\Sigma F_x = 0$
- $\Sigma F_y = 0$
- $\Sigma F_z = 0$

Rotational:

- $\Sigma T_x = 0$
- $\Sigma T_y = 0$
- $\Sigma T_z = 0$

💡 Coplanar Forces

When all forces lie in a plane (say x-y plane):

Only 3 conditions needed:

- $\Sigma F_x = 0$
- $\Sigma F_y = 0$
- $\Sigma \tau_z = 0$ (about any axis perpendicular to plane)

🎯 Partial Equilibrium

◆ Translational but Not Rotational

Example: Light rod with two equal parallel forces at ends

- $\Sigma F = 0$ (forces cancel) ✓
- But $\Sigma \tau \neq 0$ (torques add) ✗
- Result: Pure rotation without translation

◆ Rotational but Not Translational

Example: Rod with two equal opposite forces (couple)

- $\Sigma F = 0$ (forces cancel) ✓
- $\Sigma \tau \neq 0$ (both torques in same direction) ✗
- Result: Pure rotation

COUPLE: Pair of equal and opposite forces with different lines of action

Effect: Produces rotation without translation

Principle of Moments (Lever)

Lever: Light rod pivoted at fulcrum, with forces F_1 and F_2 at distances d_1 and d_2

For equilibrium:

1. $R - F_1 - F_2 = 0$ (R = reaction at fulcrum)
2. $d_1F_1 - d_2F_2 = 0$ (moments about fulcrum)

$$d_1F_1 = d_2F_2$$

Load arm × Load = Effort arm × Effort

Mechanical Advantage:

$$M.A. = F_1/F_2 = d_2/d_1$$

If $d_2 > d_1$, then $M.A. > 1$ (small effort lifts large load)

Centre of Gravity

CENTRE OF GRAVITY:

Point where total gravitational torque on body is zero

For uniform gravity field:

Total gravitational torque: $\tau_g = \sum(r_i \times m_i g)$

Since g is same for all particles: $\tau_g = (\sum m_i r_i) \times g$

For $\tau_g = 0$: $\sum m_i r_i = 0$

This defines the centre of mass!

In uniform gravity: CG coincides with CM

Finding CG Experimentally

Method: Suspend body from different points

- Vertical line through suspension point passes through CG
- Intersection of verticals from different points gives CG
- For small bodies: CG = CM

Example 6.8: Non-Uniform Bar in Equilibrium

Question: Non-uniform bar (weight W , length 2m) suspended by strings at angles 36.9° and 53.1° from vertical. Find distance d of CG from left end.

Solution:

Let R_1, R_2 be string tensions

Vertical force balance:

$$R_1 \cos(36.9^\circ) + R_2 \cos(53.1^\circ) = W$$

$$0.8R_1 + 0.6R_2 = W \dots (1)$$

Horizontal force balance:

$$R_1 \sin(36.9^\circ) = R_2 \sin(53.1^\circ)$$

$$0.6R_1 = 0.8R_2$$

$$R_1 = (4/3)R_2 \dots (2)$$

Moment about left end:

$$Wd = R_2 \cos(53.1^\circ) \times 2m$$

$$Wd = 0.6R_2 \times 2 = 1.2R_2 \dots (3)$$

From (1) and (2):

$$0.8(4/3)R_2 + 0.6R_2 = W$$

$$1.067R_2 + 0.6R_2 = W$$

$$R_2 = 0.6W$$

From (3):

$$d = 1.2R_2/W = 1.2(0.6W)/W = \mathbf{0.72 \text{ m}}$$

Example 6.9: Ladder Against Wall

Question: 3m ladder (20 kg) leans on frictionless wall, foot 1m from wall. Find reaction forces.

Solution:

$$\text{Height BC} = \sqrt{3^2 - 1^2} = 2\sqrt{2} \text{ m}$$

Let: F_1 = wall reaction (horizontal), N = normal from ground, F = friction from ground

Vertical equilibrium: $N = 20g = 196 \text{ N}$

Horizontal equilibrium: $F = F_1$

Moment about foot A:

$$2\sqrt{2} F_1 = (1/2) \times 20g \times 1$$

$$2\sqrt{2} F_1 = 10g = 98$$

$$F_1 = 98/(2\sqrt{2}) = \mathbf{34.6 \text{ N}}$$

Therefore: $F = \mathbf{34.6 \text{ N}}$

Resultant floor reaction: $F_2 = \sqrt{N^2 + F^2} = \sqrt{196^2 + 34.6^2} \approx \mathbf{199 \text{ N}}$

Angle with horizontal: $\alpha = \tan^{-1}(N/F) = \tan^{-1}(4\sqrt{2}) \approx \mathbf{80^\circ}$

6.9 MOMENT OF INERTIA

The rotational analogue of mass - a measure of how mass is distributed with respect to the axis of rotation.

Kinetic energy of rotating body:

For particle i at distance r_i from axis:

$$K_i = (1/2)m_i v_i^2 = (1/2)m_i (\omega r_i)^2 = (1/2)m_i r_i^2 \omega^2$$

Total KE:

$$K = \sum K_i = \sum (1/2)m_i r_i^2 \omega^2 = (1/2)\omega^2 \sum m_i r_i^2$$

Define moment of inertia I:

$$\mathbf{I = \sum m_i r_i^2}$$

where r_i = perpendicular distance of particle i from axis

Therefore:

$$\mathbf{K = (1/2)I\omega^2}$$

☀ Analogy with Linear Motion

| Linear Motion | Rotational Motion |
|-----------------|---------------------------|
| Mass m | Moment of inertia I |
| Velocity v | Angular velocity ω |
| $K = (1/2)mv^2$ | $K = (1/2)I\omega^2$ |

I is the rotational analogue of mass!

🎯 Simple Examples

📄 Ring About Central Axis

Ring of radius R , mass M , rotating about axis through centre perpendicular to plane

All particles at distance R from axis:

$$I = \sum m_i R^2 = R^2 \sum m_i = \mathbf{MR^2}$$

Two Masses on Rod

Light rod of length l with two masses $M/2$ at each end, rotating about centre perpendicular to rod

Each mass at distance $l/2$:

$$I = (M/2)(l/2)^2 + (M/2)(l/2)^2$$

$$= 2 \times (M/2) \times (l^2/4)$$

$$= \mathbf{Ml^2/4}$$

Common Moments of Inertia

| Body | Axis | Moment of Inertia |
|---------------------|------------------------|-------------------|
| Thin ring (R) | Central, \perp plane | MR^2 |
| Thin ring (R) | Diameter | $MR^2/2$ |
| Thin rod (L) | \perp at midpoint | $ML^2/12$ |
| Circular disc (R) | Central, \perp plane | $MR^2/2$ |
| Circular disc (R) | Diameter | $MR^2/4$ |
| Hollow cylinder (R) | Cylinder axis | MR^2 |
| Solid cylinder (R) | Cylinder axis | $MR^2/2$ |
| Solid sphere (R) | Diameter | $2MR^2/5$ |

Radius of Gyration

$$I = Mk^2$$

where k = radius of gyration

Physical Meaning

Radius of gyration k : Distance from axis at which entire mass could be concentrated to give same moment of inertia

Examples:

- Rod about midpoint: $k = L/\sqrt{12}$
- Disc about centre: $k = R/\sqrt{2}$
- Sphere about diameter: $k = R\sqrt{(2/5)}$

Properties of Moment of Inertia

- ✓ **Dimensions:** $[M L^2]$
- ✓ **SI Units:** $\text{kg}\cdot\text{m}^2$
- ✓ Depends on mass, shape, size, AND axis position/orientation
- ✓ NOT a fixed property of the body (unlike mass)
- ✓ Measure of rotational inertia

Application: Flywheel

Flywheel

Disc with large moment of inertia used in machines (engines, automobiles)

Purpose:

- Resists sudden changes in rotational speed
- Ensures smooth, gradual speed changes
- Prevents jerky motion
- Stores rotational kinetic energy

6.10 KINEMATICS OF ROTATIONAL MOTION

Analogous to kinematics of linear motion, but for rotation about a fixed axis.

Kinematic Variables

| Linear Motion | Rotational Motion |
|--------------------------|--|
| Displacement s | Angular displacement θ |
| Velocity $v = ds/dt$ | Angular velocity $\omega = d\theta/dt$ |
| Acceleration $a = dv/dt$ | Angular acceleration $\alpha = d\omega/dt$ |

Equations for Uniform Angular Acceleration

Analogous to $v = v_0 + at$:

$$\omega = \omega_0 + \alpha t$$

Analogous to $s = s_0 + v_0t + (1/2)at^2$:

$$\theta = \theta_0 + \omega_0t + (1/2)\alpha t^2$$

Analogous to $v^2 = v_0^2 + 2as$:

$$\omega^2 = \omega_0^2 + 2\alpha(\theta - \theta_0)$$

 **Example 6.10: Deriving $\omega = \omega_0 + \alpha t$**

Given: Angular acceleration $\alpha = \text{constant}$

From definition: $\alpha = d\omega/dt$

Integrating:

$$\omega = \int \alpha dt = \alpha t + c$$

Initial condition: At $t = 0$, $\omega = \omega_0$

Therefore: $c = \omega_0$

Result: $\omega = \omega_0 + \alpha t \checkmark$

Example 6.11: Motor Wheel Acceleration

Question: Angular speed increases from 1200 rpm to 3120 rpm in 16 seconds.

- (a) Find angular acceleration (assume uniform)
- (b) How many revolutions does engine make?

Solution:

(a) Convert to rad/s:

$$\omega_0 = 1200 \text{ rpm} = (2\pi \times 1200)/60 = 40\pi \text{ rad/s}$$

$$\omega = 3120 \text{ rpm} = (2\pi \times 3120)/60 = 104\pi \text{ rad/s}$$

Angular acceleration:

$$\alpha = (\omega - \omega_0)/t = (104\pi - 40\pi)/16$$

$$= 64\pi/16 = \mathbf{4\pi \text{ rad/s}^2}$$

(b) Angular displacement:

$$\theta = \omega_0 t + (1/2)\alpha t^2$$

$$= 40\pi(16) + (1/2)(4\pi)(16)^2$$

$$= 640\pi + 512\pi = 1152\pi \text{ rad}$$

Number of revolutions:

$$N = \theta/(2\pi) = 1152\pi/(2\pi) = \mathbf{576 \text{ revolutions}}$$

6.11 DYNAMICS OF ROTATIONAL MOTION

Establishing Newton's Second Law for rotation: $\tau = I\alpha$

Work Done by Torque

For particle at distance r from axis, force F tangential:

Work done: $dW = F \cdot ds = F(r d\theta)$

Torque magnitude: $\tau = rF$

Therefore:

$$\mathbf{dW = \tau d\theta}$$

This is analogous to $dW = F ds$ in linear motion

Power

Power = rate of doing work:

$P = dW/dt = \tau(d\theta/dt)$

$$\mathbf{P = \tau\omega}$$

Analogous to $P = Fv$ in linear motion

Torque and Angular Acceleration

Work-energy theorem approach:

Rate of work = Rate of increase of KE

$$\tau\omega = d/dt[(1/2)I\omega^2] = I\omega(d\omega/dt)$$

Since $\alpha = d\omega/dt$:

$$\tau\omega = I\omega\alpha$$

$$\tau = I\alpha$$

Newton's Second Law for rotation about fixed axis!

Rotational Analogue of $F = ma$

| Linear Motion | Rotational Motion |
|------------------------------|---|
| $F = ma$ | $\tau = I\alpha$ |
| Force produces acceleration | Torque produces angular acceleration |
| $a \propto F, a \propto 1/m$ | $\alpha \propto \tau, \alpha \propto 1/I$ |

Example 6.12: Flywheel with Cord

Question: Cord wound around rim of flywheel ($M = 20 \text{ kg}$, $R = 0.2 \text{ m}$). Steady pull $F = 25 \text{ N}$.

- (a) Compute angular acceleration
- (b) Work done when 2m of cord unwound
- (c) Kinetic energy at this point (starts from rest)
- (d) Compare (b) and (c)

Solution:

(a) Using $\tau = I\alpha$:

$$\text{Torque: } \tau = FR = 25 \times 0.2 = 5.0 \text{ N}\cdot\text{m}$$

$$\text{Moment of inertia: } I = MR^2/2 = 20(0.2)^2/2 = 0.4 \text{ kg}\cdot\text{m}^2$$

$$\alpha = \tau/I = 5.0/0.4 = \mathbf{12.5 \text{ rad/s}^2}$$

(b) Work done:

$$W = F \times \text{distance} = 25 \times 2 = \mathbf{50 \text{ J}}$$

(c) Kinetic energy:

$$\text{Angular displacement: } \theta = (\text{length unwound})/R = 2/0.2 = 10 \text{ rad}$$

$$\text{Using } \omega^2 = \omega_0^2 + 2\alpha\theta: \omega^2 = 0 + 2(12.5)(10) = 250 \text{ rad}^2/\text{s}^2$$

$$K = (1/2)I\omega^2 = (1/2)(0.4)(250) = \mathbf{50 \text{ J}}$$

(d) Work done = KE gained (no friction - energy conserved) ✓

6.12 ANGULAR MOMENTUM FOR ROTATION ABOUT FIXED AXIS

Angular Momentum Components

For particle i rotating about z -axis:

Angular momentum: $l_i = r_i \times p_i$

Component along axis (z -component):

$$l_{iz} = m_i r_{i\perp}^2 \omega$$

Total for symmetric body:

$$L_z = \sum m_i r_{i\perp}^2 \omega = I \omega$$

$$\mathbf{L} = I \boldsymbol{\omega} \text{ (for symmetric bodies)}$$

Important Notes

- For symmetric bodies rotating about symmetry axis: **L and ω are parallel**
- For asymmetric bodies: L and ω may not be parallel
- Compare with linear motion: p and v are ALWAYS parallel

Relation with Torque

Differentiating $L_z = I\omega$:

$$dL_z/dt = I(d\omega/dt) = I\alpha$$

Since $\tau = I\alpha$:

$$dL/dt = \tau$$

Conservation of Angular Momentum

If $\tau_{\text{ext}} = 0$:

$$L = I\omega = \text{constant}$$

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Demonstration: Rotating Chair

Setup: Person sits on swivel chair (frictionless), arms outstretched, rotating

Action: Person brings arms closer to body

Observation: Angular speed INCREASES

Explanation:

- No external torque (frictionless)
- Therefore: $I\omega = \text{constant}$
- Arms in $\rightarrow I$ decreases
- To conserve L : ω must increase!
- Arms out \rightarrow reverse effect

Applications: Divers, acrobats, ice skaters, dancers performing pirouettes

SUMMARY TABLE

Linear vs Rotational Motion

| Quantity | Linear Motion | Rotational Motion |
|---------------------------------|---|---|
| Displacement | s | θ |
| Velocity | $v = ds/dt$ | $\omega = d\theta/dt$ |
| Acceleration | $a = dv/dt$ | $\alpha = d\omega/dt$ |
| Mass/Inertia | m | $I = \Sigma mr^2$ |
| Force/Torque | F | $\tau = r \times F$ |
| Newton's 2nd Law | $F = ma$ | $\tau = I\alpha$ |
| Work | $dW = F ds$ | $dW = \tau d\theta$ |
| Kinetic Energy | $K = (1/2)mv^2$ | $K = (1/2)I\omega^2$ |
| Power | $P = Fv$ | $P = \tau\omega$ |
| Momentum | $p = mv$ | $L = I\omega$ |
| Equations (const. accel) | $v = v_0 + at$ $s = v_0t + (1/2)at^2$ $v^2 = v_0^2 + 2as$ | $\omega = \omega_0 + \alpha t$ $\theta = \omega_0t + (1/2)\alpha t^2$ $\omega^2 = \omega_0^2 + 2\alpha\theta$ |

PRACTICE QUESTIONS

Section A: Multiple Choice Questions (1 mark each)

Q1. The moment of inertia of a uniform circular disc about an axis perpendicular to its plane and passing through its centre is:

- (a) MR^2
- (b) $MR^2/2$ ✓
- (c) $MR^2/4$
- (d) $2MR^2/5$

Answer: (b) For uniform circular disc about central perpendicular axis: $I = MR^2/2$

Q2. If no external torque acts on a body, which quantity will be conserved?

- (a) Force
- (b) Linear momentum
- (c) Angular momentum ✓
- (d) Impulse

Answer: (c) When $\tau_{\text{ext}} = 0$, angular momentum L is conserved

Q3. For a rigid body in mechanical equilibrium:

- (a) Only $\Sigma F = 0$
- (b) Only $\Sigma \tau = 0$
- (c) Both $\Sigma F = 0$ and $\Sigma \tau = 0$ ✓
- (d) Neither condition needed

Answer: (c) Both translational and rotational equilibrium required

Section B: Short Answer Questions (2-3 marks each)

Q4. State and prove the principle of moments for a lever.

Answer:

Principle of Moments: For a lever in equilibrium:

Load arm \times Load = Effort arm \times Effort

$$d_1F_1 = d_2F_2$$

Proof: For rotational equilibrium about fulcrum, sum of moments = 0. Taking clockwise moments as positive and anticlockwise as negative: $d_1F_1 - d_2F_2 = 0$, giving $d_1F_1 = d_2F_2$

Q5. Distinguish between centre of mass and centre of gravity.

Answer:

- **Centre of Mass:** Point where entire mass can be assumed concentrated; defined from mass distribution only
- **Centre of Gravity:** Point where total gravitational torque is zero
- **Relationship:** In uniform gravitational field, CG coincides with CM. For extended bodies in non-uniform field, they may differ

Section C: Long Answer Questions (5 marks each)

Q6. Derive the relation $\tau = I\alpha$ for a rigid body rotating about a fixed axis.

Answer:

Step 1: Consider work done by torque

For particle at distance r : $dW = F \cdot ds = F(r d\theta)$

Since $\tau = rF$: $dW = \tau d\theta$

Step 2: Power (rate of work)

$$P = dW/dt = \tau(d\theta/dt) = \tau\omega$$

Step 3: Equate to rate of KE increase

$$dK/dt = d/dt[(1/2)I\omega^2] = (1/2)I \cdot 2\omega(d\omega/dt) = I\omega(d\omega/dt) = I\omega\alpha$$

Step 4: Therefore

$$\tau\omega = I\omega\alpha$$

$$\tau = I\alpha \text{ (dividing by } \omega \text{)}$$

This is Newton's Second Law for rotational motion about a fixed axis.



EXAM PREPARATION TIPS

🌟 Master These Concepts:

- 🌟 Centre of mass definition and calculation for different systems
- 🌟 Motion of centre of mass: $MA = F_{\text{ext}}$ (no internal forces needed)
- 🌟 Vector product properties and applications
- 🌟 $v = \omega \times r$ relationship between linear and angular velocity
- 🌟 Torque $\tau = r \times F$ and angular momentum $l = r \times p$
- 🌟 Equilibrium conditions: $\Sigma F = 0$ AND $\Sigma \tau = 0$
- 🌟 Moment of inertia: definition, physical meaning, common values
- 🌟 Rotational kinematics equations (analogous to linear)
- 🌟 $\tau = I\alpha$ (rotational Newton's Second Law)
- 🌟 Conservation laws: linear momentum and angular momentum

💪 KEY TO SUCCESS:

"Master the Linear-Rotational Analogy - It's the Foundation of Rotational Mechanics!"

Must Practice:

- All derivations (especially $\tau = I\alpha$, conservation laws)
- 20 numerical problems on centre of mass calculations
- 15 problems on moment of inertia and rotational KE
- 15 equilibrium problems (levers, ladders, suspended bodies)
- 10 vector product calculations
- 10 conservation of angular momentum problems
- All NCERT examples and exercises

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Study Material Information

This comprehensive study material on **Systems of Particles and Rotational Motion** has been prepared following the latest CBSE curriculum and examination pattern for Class 11 Physics. The content includes detailed explanations of centre of mass, motion of centre of mass, linear momentum conservation, vector products, angular velocity, torque, angular momentum, equilibrium conditions, moment of inertia, rotational kinematics and dynamics, complete derivations from NCERT, and practice questions aligned with current board exam format.

Key Features of This Material:

- Complete chapter coverage with crystal-clear concepts
- Centre of mass: definition, calculation, and physical significance
- Motion of CM and Newton's Second Law for systems
- Conservation of linear momentum with applications
- Vector product: definition, properties, component form
- Angular velocity and its relation with linear velocity
- Torque and angular momentum: definitions and applications
- Equilibrium of rigid bodies: translational and rotational
- Principle of moments and centre of gravity
- Moment of inertia: concept, examples, radius of gyration
- Rotational kinematics equations (uniform angular acceleration)
- Dynamics: work, power, $\tau = I\alpha$ derivation
- Conservation of angular momentum with real-life examples
- All NCERT solved examples (6.1-6.12) with detailed solutions
- Multiple choice and descriptive practice questions
- Comprehensive linear-rotational analogy table
- Exam-focused tips and strategies

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