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INTRODUCTION TO TRIGONOMETRY

Complete Study Material for CBSE Class 10 (2025-

Chapter 8

🌟 FASCINATING FACTS ABOUT TRIGONOMETRY

Did You Know?

📜 **Ancient Origins:** The word 'trigonometry' comes from Greek: 'tri' (three) + 'gon' (sides) + 'metron' (measure). It literally means "measurement of triangles"!

IN **Indian Contribution:** Aryabhata (476-550 CE) was the first to use the concept of sine (jya) in his work Aryabhatiyam. The word "sine" comes from the Sanskrit word "jya"!

⭐ **Astronomy Connection:** Early astronomers used trigonometry to calculate distances to stars and planets. Ancient Egyptians and Babylonians used basic trigonometry to build pyramids and study celestial bodies!

🏗 **Architecture & Engineering:** The world's tallest buildings, longest bridges, and most complex structures are designed using trigonometry. Engineers use it to calculate forces, angles, and stability!

📱 **Modern Technology:** Your GPS, smartphone, computer graphics, video games, and even music synthesis use trigonometry! Every 3D animation you see uses trigonometric calculations!

🌊 **Wave Motion:** Sound waves, light waves, ocean waves, and electromagnetic waves are all described using trigonometric functions (sine and cosine waves)!

🎵 **Music & Sound:** Musical notes are sine waves! When you hear music, your ears are detecting trigonometric functions. Sound engineers use trigonometry to create perfect acoustics!

🚀 **Space Missions:** NASA uses trigonometry to calculate rocket trajectories, satellite positions, and planetary orbits. The Mars rover landing used complex trigonometric calculations!

🎨 **Art & Design:** Artists use trigonometry to create perspective in paintings. Computer graphics designers use it to create 3D models and animations!

📡 **Communication:** Radio waves, WiFi, Bluetooth, and all wireless communication technologies are based on trigonometric principles!

📋 CHAPTER OVERVIEW

Chapter	Introduction to Trigonometry (Chapter 8)
Weightage	12 marks (As per CBSE marking scheme 2025-26)
Difficulty Level	Medium to High
Expected Questions	2-3 questions (2 marks, 3 marks, or 5 marks)
Time Required	15-20 minutes in exam
Prerequisites	Right triangles, Pythagoras Theorem, Ratios, Square roots

Topics Covered:

1. **Trigonometric Ratios - Definition**
2. **Relationships between Ratios**
3. **Trigonometric Ratios of Standard Angles (0°, 30°, 45°, 60°, 90°)**

4. **Trigonometric Identities**
5. **Applications in Problem Solving**

⚠️ IMPORTANT NOTE - CBSE 2025-26:

This chapter is HIGH WEIGHTAGE - 12 marks!

Focus on: **Trigonometric Ratios, Standard Angles Table, and Trigonometric Identities**

Must Memorize: All 6 trigonometric ratios and values for 0°, 30°, 45°, 60°, 90°

🎯 SECTION 1: UNDERSTANDING RIGHT TRIANGLES

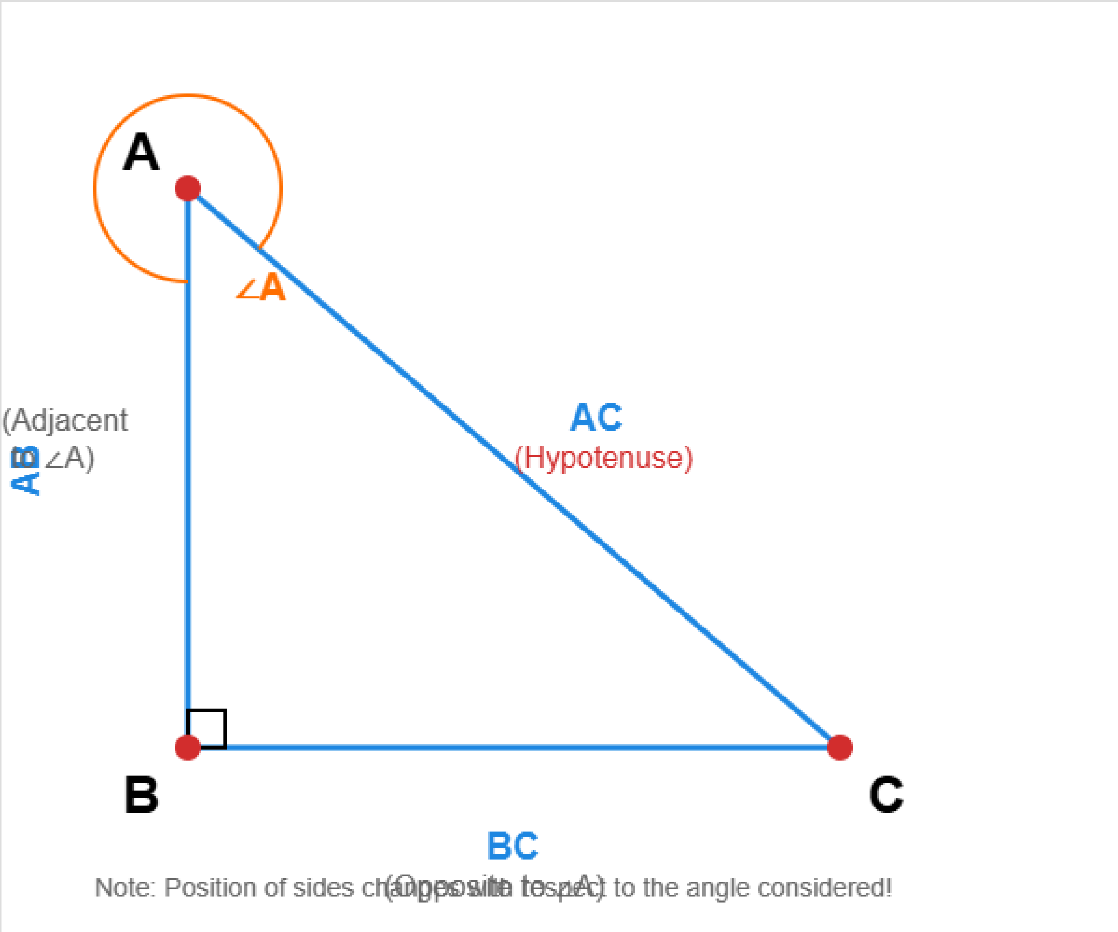
What is a Right Triangle?

Right Triangle: A triangle with one angle equal to 90°

- **Hypotenuse:** The longest side (opposite to 90° angle)
- **Perpendicular (Opposite):** Side opposite to the angle being considered
- **Base (Adjacent):** Side next to the angle being considered (not hypotenuse)

Important: The position of perpendicular and base changes based on which angle you're considering!

Right Triangle ABC with Angle A



🔑 Key Understanding:

- ✓ **Hypotenuse is ALWAYS opposite the 90° angle**
- ✓ **Opposite side changes** based on which acute angle you consider
- ✓ **Adjacent side changes** based on which acute angle you consider
- ✓ For angle A: Opposite = BC, Adjacent = AB
- ✓ For angle C: Opposite = AB, Adjacent = BC

🎯 SECTION 2: TRIGONOMETRIC RATIOS

THE SIX TRIGONOMETRIC RATIOS

For angle A in a right triangle ABC (right-angled at B):

sin A = Perpendicular/Hypotenuse = BC/AC

cos A = Base/Hypotenuse = AB/AC

tan A = Perpendicular/Base = BC/AB

Reciprocal Ratios:

cosec A = 1/sin A = Hypotenuse/Perpendicular = AC/BC

sec A = 1/cos A = Hypotenuse/Base = AB/AC

cot A = 1/tan A = Base/Perpendicular = AB/BC

📝 Memory Trick: "Some People Have - Curly Brown Hair - Through Proper Brushing"

Some **P**eople **H**ave → **S**in = **P**erpendicular/**H**ypotenuse

Curly **B**rown **H**air → **C**os = **B**ase/**H**ypotenuse

Through **P**roper **B**rushing → **T**an = **P**erpendicular/**B**ase

Important Relationships

RECIPROCAL RELATIONSHIPS

cosec A = 1/sin A   sec A = 1/cos A   cot A = 1/tan A

sin A = 1/cosec A   cos A = 1/sec A   tan A = 1/cot A

QUOTIENT RELATIONSHIPS

tan A = sin A/cos A   cot A = cos A/sin A

💡 **Important Notes:**

- ✓ **sin A and cos A are always ≤ 1** (since hypotenuse is longest)
- ✓ **sec A and cosec A are always ≥ 1**
- ✓ tan A and cot A can be any value
- ✓ $\sin^2 A$ means $(\sin A)^2$, NOT $\sin(A^2)$
- ✓ These ratios depend ONLY on the angle, not on the size of triangle

TYPE 1: Finding All Ratios from One Ratio

Example 1: If sin A = 3/5, find all other trigonometric ratios of angle A.

Solution:

Given: $\sin A = 3/5$

This means: Perpendicular/Hypotenuse = 3/5

So, Perpendicular = 3k, Hypotenuse = 5k (where $k > 0$)

Step 1: Find Base using Pythagoras theorem

$\text{Base}^2 = \text{Hypotenuse}^2 - \text{Perpendicular}^2$

$\text{Base}^2 = (5k)^2 - (3k)^2$

$\text{Base}^2 = 25k^2 - 9k^2$

$\text{Base}^2 = 16k^2$

Base = 4k

Step 2: Find all ratios

$\sin A = 3/5$ (given)

$\cos A$ = Base/Hypotenuse = 4k/5k = **4/5**

$\tan A$ = Perpendicular/Base = 3k/4k = **3/4**

$\text{cosec } A = 1/\sin A$ = **5/3**

$\sec A = 1/\cos A$ = **5/4**

$\cot A = 1/\tan A$ = **4/3**

Answer: cos A = 4/5, tan A = 3/4, cosec A = 5/3, sec A = 5/4, cot A = 4/3

Example 2: If $\tan \theta = 12/5$, find $\sin \theta$ and $\cos \theta$.

Solution:

Given: $\tan \theta = 12/5$

This means: Perpendicular/Base = 12/5

So, Perpendicular = 12k, Base = 5k

Find Hypotenuse:

$\text{Hypotenuse}^2 = (12k)^2 + (5k)^2$

$\text{Hypotenuse}^2 = 144k^2 + 25k^2$

$\text{Hypotenuse}^2 = 169k^2$

Hypotenuse = 13k

Therefore:

$\sin \theta$ = Perpendicular/Hypotenuse = 12k/13k = **12/13**

$\cos \theta$ = Base/Hypotenuse = 5k/13k = **5/13**

Answer: $\sin \theta = 12/13$, $\cos \theta = 5/13$

Example 3: If cot A = 7/8, evaluate: (i) (1+sin A)(1-sin A)/(1+cos A)(1-cos A), (ii) $\cot^2 A$

Solution:

Given: cot A = 7/8

Base = 7k, Perpendicular = 8k

Hypotenuse = $\sqrt{(7k)^2 + (8k)^2} = \sqrt{49k^2 + 64k^2} = \sqrt{113k^2} = k\sqrt{113}$

sin A = $8k/(k\sqrt{113}) = 8/\sqrt{113}$

cos A = $7k/(k\sqrt{113}) = 7/\sqrt{113}$

(i) (1+sin A)(1-sin A)/(1+cos A)(1-cos A)

Using $(a+b)(a-b) = a^2 - b^2$:

Numerator = $1 - \sin^2 A$

Denominator = $1 - \cos^2 A$

= $(1 - \sin^2 A)/(1 - \cos^2 A)$

= $\cos^2 A/\sin^2 A$ [Using identity: $\sin^2 A + \cos^2 A = 1$]

= $\cot^2 A$

= $(7/8)^2$

= **49/64**

(ii) $\cot^2 A$

$\cot^2 A = (7/8)^2 =$ **49/64**

Answer: (i) 49/64, (ii) 49/64

🎯 SECTION 3: TRIGONOMETRIC RATIOS OF STANDARD ANGLES

Why These Angles?

The angles 0°, 30°, 45°, 60°, and 90° are called **standard angles** because:

- ✓ They can be constructed using compass and ruler
- ✓ Their trigonometric ratios have exact values (not approximations)
- ✓ They are most commonly used in applications
- ✓ They form the basis for calculating other angles

Derivation of Standard Angles

1. Trigonometric Ratios of 45°

Consider an isosceles right triangle:

In △ABC, ∠B = 90°, ∠A = ∠C = 45°

Therefore, AB = BC (sides opposite equal angles)

Let AB = BC = a

By Pythagoras theorem:

$AC^2 = AB^2 + BC^2 = a^2 + a^2 = 2a^2$

$AC = a\sqrt{2}$

For angle 45°:

```
sin 45° = BC/AC = a/(a√2) = 1/√2 = √2/2
cos 45° = AB/AC = a/(a√2) = 1/√2 = √2/2
tan 45° = BC/AB = a/a = 1

cosec 45° = √2
sec 45° = √2
cot 45° = 1
```

2. Trigonometric Ratios of 30° and 60°

Consider an equilateral triangle:

△ABC where AB = BC = CA = 2a, and all angles = 60°

Draw altitude AD from A to BC

AD bisects BC, so BD = DC = a

Also, ∠BAD = ∠CAD = 30°

In △ABD: AB = 2a, BD = a

By Pythagoras: $AD^2 = AB^2 - BD^2 = 4a^2 - a^2 = 3a^2$

AD = a√3

For angle 30°:

```
sin 30° = BD/AB = a/(2a) = 1/2
cos 30° = AD/AB = (a√3)/(2a) = √3/2
tan 30° = BD/AD = a/(a√3) = 1/√3 = √3/3

cosec 30° = 2
sec 30° = 2/√3 = 2√3/3
cot 30° = √3
```

For angle 60°:

```
sin 60° = AD/AB = (a√3)/(2a) = √3/2
cos 60° = BD/AB = a/(2a) = 1/2
tan 60° = AD/BD = (a√3)/a = √3

cosec 60° = 2/√3 = 2√3/3
sec 60° = 2
cot 60° = 1/√3 = √3/3
```

3. Trigonometric Ratios of 0° and 90°

Understanding through limiting cases:

As angle A → 0°:

- Perpendicular → 0
- Base → Hypotenuse

Therefore: $\sin 0° = 0$, $\cos 0° = 1$, $\tan 0° = 0$

As angle A → 90°:

- Perpendicular → Hypotenuse
- Base → 0

Therefore: $\sin 90° = 1$, $\cos 90° = 0$, $\tan 90°$ = not defined

📊 TRIGONOMETRIC TABLE - MUST MEMORIZE!

Angle (A)	0°	30°	45°	60°	90°
sin A	0	1/√2	1/√2	√3/2	1
cos A	1	√3/2	1/√2	1/2	0
tan A	0	1/√3	1	√3	Not defined
cosec A	Not defined	2	√2	2/√3	1
sec A	1	2/√3	√2	2	Not defined
cot A	Not defined	√3	1	1/√3	0

🎯 EASY TRICKS TO REMEMBER THE TABLE:

Method 1: Fraction Pattern for sin and cos

For sin: $0°, 30°, 45°, 60°, 90° \rightarrow \sqrt{0}/2, \sqrt{1}/2, \sqrt{2}/2, \sqrt{3}/2, \sqrt{4}/2$

$= 0, 1/2, 1/\sqrt{2}, \sqrt{3}/2, 1$

For cos: REVERSE of sin!

$90°, 60°, 45°, 30°, 0° \rightarrow \sqrt{4}/2, \sqrt{3}/2, \sqrt{2}/2, \sqrt{1}/2, \sqrt{0}/2$

$= 1, \sqrt{3}/2, 1/\sqrt{2}, 1/2, 0$

Method 2: Hand Trick

For sin: Count fingers from 0° (0) to 90° (4), divide by 2, take square root!

- $0° \rightarrow \sqrt{(0/4)} = 0$
- $30° \rightarrow \sqrt{(1/4)} = 1/2$
- $45° \rightarrow \sqrt{(2/4)} = 1/\sqrt{2}$
- $60° \rightarrow \sqrt{(3/4)} = \sqrt{3}/2$
- $90° \rightarrow \sqrt{(4/4)} = 1$

Method 3: Observations

- ✓ $\sin 30° = \cos 60° = 1/2$
- ✓ $\sin 60° = \cos 30° = \sqrt{3}/2$
- ✓ $\sin 45° = \cos 45° = 1/\sqrt{2}$
- ✓ $\tan 45° = \cot 45° = 1$
- ✓ $\tan 30° = \cot 60° = 1/\sqrt{3}$
- ✓ $\tan 60° = \cot 30° = \sqrt{3}$

Applications of Standard Angles

Example 4: Evaluate: sin 60° cos 30° + sin 30° cos 60°

Solution:

$\sin 60° \cos 30° + \sin 30° \cos 60°$

$= (\sqrt{3}/2)(\sqrt{3}/2) + (1/2)(1/2)$

$= 3/4 + 1/4$

$=$ **4/4 = 1**

Answer: 1

Example 5: Evaluate: 2 $\tan^2 45° + \cos^2 30° - \sin^2 60°$

Solution:

$2 \tan^2 45° + \cos^2 30° - \sin^2 60°$

$= 2(1)^2 + (\sqrt{3}/2)^2 - (\sqrt{3}/2)^2$

$= 2(1) + 3/4 - 3/4$

$= 2 + 0$

$=$ **2**

Answer: 2

Example 6: If sin(A-B) = 1/2, cos(A+B) = 1/2, 0° < A+B ≤ 90°, A > B, find A and B.

Solution:

Given: sin(A-B) = 1/2

From table: sin 30° = 1/2

Therefore: **A - B = 30°** ... (1)

Given: cos(A+B) = 1/2

From table: cos 60° = 1/2

Therefore: **A + B = 60°** ... (2)

Adding equations (1) and (2):

(A-B) + (A+B) = 30° + 60°

2A = 90°

A = 45°

Substituting in (2):

45° + B = 60°

B = 15°

Answer: A = 45°, B = 15°

Example 7: In △ABC, right-angled at B, AB = 5 cm and ∠ACB = 30°. Find BC and AC.

Solution:

Given: AB = 5 cm, ∠C = 30°, ∠B = 90°

For angle C = 30°:

$\tan 30° = AB/BC$

$1/\sqrt{3} = 5/BC$

$BC = 5\sqrt{3}$ cm

Now, using Pythagoras theorem:

$AC^2 = AB^2 + BC^2$

$AC^2 = 5^2 + (5\sqrt{3})^2$

$AC^2 = 25 + 75$

$AC^2 = 100$

$AC = 10$ cm

Alternative method using sin 30°:

$\sin 30° = AB/AC$

$1/2 = 5/AC$

$AC = 10$ cm ✓

Answer: BC = $5\sqrt{3}$ cm, AC = 10 cm

🎯 SECTION 4: TRIGONOMETRIC IDENTITIES

What is an Identity?

An **identity** is an equation that is TRUE for ALL values of the variable.

A **trigonometric identity** is an equation involving trigonometric ratios that is true for all values of the angle(s) for which the ratios are defined.

THE THREE FUNDAMENTAL TRIGONOMETRIC IDENTITIES

Identity 1: $\sin^2 A + \cos^2 A = 1$

Identity 2: $1 + \tan^2 A = \sec^2 A$

Identity 3: $1 + \cot^2 A = \operatorname{cosec}^2 A$

Proof of Identity 1: $\sin^2 A + \cos^2 A = 1$

Proof:

Consider right triangle ABC with $\angle B = 90°$

By Pythagoras theorem:

$AB^2 + BC^2 = AC^2$

Dividing both sides by AC^2:

$AB^2/AC^2 + BC^2/AC^2 = AC^2/AC^2$

$(AB/AC)^2 + (BC/AC)^2 = 1$

But $AB/AC = \cos A$ and $BC/AC = \sin A$

Therefore: $\mathbf{\cos^2 A + \sin^2 A = 1}$

Or: $\mathbf{\sin^2 A + \cos^2 A = 1}$ ✓

Proof of Identity 2: $1 + \tan^2 A = \sec^2 A$

Proof:

Start with: $AB^2 + BC^2 = AC^2$

Dividing both sides by AB^2:

$AB^2/AB^2 + BC^2/AB^2 = AC^2/AB^2$

$1 + (BC/AB)^2 = (AC/AB)^2$

But $BC/AB = \tan A$ and $AC/AB = \sec A$

Therefore: **$1 + \tan^2 A = \sec^2 A$** ✓

Proof of Identity 3: $1 + \cot^2 A = \operatorname{cosec}^2 A$

Proof:

Start with: $AB^2 + BC^2 = AC^2$

Dividing both sides by BC^2:

$AB^2/BC^2 + BC^2/BC^2 = AC^2/BC^2$

$(AB/BC)^2 + 1 = (AC/BC)^2$

But $AB/BC = \cot A$ and $AC/BC = \operatorname{cosec} A$

Therefore: **$\cot^2 A + 1 = \operatorname{cosec}^2 A$**

Or: **$1 + \cot^2 A = \operatorname{cosec}^2 A$** ✓

💡 DERIVED FORMS - Very Useful!

From Identity 1: $\sin^2 A + \cos^2 A = 1$

- $\sin^2 A = 1 - \cos^2 A$
- $\cos^2 A = 1 - \sin^2 A$
- $\sin A = \sqrt{(1 - \cos^2 A)}$
- $\cos A = \sqrt{(1 - \sin^2 A)}$

From Identity 2: $1 + \tan^2 A = \sec^2 A$

- $\tan^2 A = \sec^2 A - 1$
- $\sec^2 A - \tan^2 A = 1$
- $\tan A = \sqrt{(\sec^2 A - 1)}$
- $\sec A = \sqrt{(1 + \tan^2 A)}$

From Identity 3: $1 + \cot^2 A = \text{cosec}^2 A$

- $\cot^2 A = \text{cosec}^2 A - 1$
- $\text{cosec}^2 A - \cot^2 A = 1$
- $\cot A = \sqrt{(\text{cosec}^2 A - 1)}$
- $\text{cosec}\, A = \sqrt{(1 + \cot^2 A)}$

TYPE 1: Proving Identities

Example 8: Prove that: $\sec A\,(1 - \sin A)(\sec A + \tan A) = 1$

Solution:

LHS $= \sec A\,(1 - \sin A)(\sec A + \tan A)$

$= \sec A\,(1 - \sin A) \times (\sec A + \tan A)$

$= (1/\cos A)(1 - \sin A) \times (1/\cos A + \sin A/\cos A)$

$= (1/\cos A)(1 - \sin A) \times (1 + \sin A)/\cos A$

$= (1/\cos^2 A) \times (1 - \sin A)(1 + \sin A)$

$= (1/\cos^2 A) \times (1 - \sin^2 A)$ [Using $(a-b)(a+b) = a^2 - b^2$]

$= (1/\cos^2 A) \times \cos^2 A$ [Using $\sin^2 A + \cos^2 A = 1$]

$= 1$

= RHS ✓

Hence Proved

Example 9: Prove that: $(\text{cosec}\ \theta - \cot\theta)^2 = (1-\cos\theta)/(1+\cos\theta)$

Solution:

LHS = $(\text{cosec}\ \theta - \cot\theta)^2$

$= (1/\sin\theta - \cos\theta/\sin\theta)^2$

$= [(1 - \cos\theta)/\sin\theta]^2$

$= (1 - \cos\theta)^2/\sin^2\theta$

$= (1 - \cos\theta)^2/(1 - \cos^2\theta)$ [Using $\sin^2\theta = 1 - \cos^2\theta$]

$= (1 - \cos\theta)^2/[(1 - \cos\theta)(1 + \cos\theta)]$

$= (1 - \cos\theta)/(1 + \cos\theta)$

= RHS ✓

Hence Proved

Example 10: Prove that: $(\sin A + \cos A)^2 + (\sin A - \cos A)^2 = 2$

Solution:

LHS = $(\sin A + \cos A)^2 + (\sin A - \cos A)^2$

Expanding using $(a+b)^2 = a^2 + 2ab + b^2$ and $(a-b)^2 = a^2 - 2ab + b^2$:

$= (\sin^2 A + 2\sin A \cos A + \cos^2 A) + (\sin^2 A - 2\sin A \cos A + \cos^2 A)$

$= \sin^2 A + \cos^2 A + \sin^2 A + \cos^2 A + 2\sin A \cos A - 2\sin A \cos A$

$= 2\sin^2 A + 2\cos^2 A$

$= 2(\sin^2 A + \cos^2 A)$

$= 2(1)$ [Using $\sin^2 A + \cos^2 A = 1$]

$= 2$

= RHS ✓

Hence Proved

TYPE 2: Simplification Using Identities

Example 11: Express sin A, sec A and tan A in terms of cot A.

Solution:

We know: $1 + \cot^2 A = \text{cosec}^2 A$

Finding sin A:

$\text{cosec}^2 A = 1 + \cot^2 A$

$1/\sin^2 A = 1 + \cot^2 A$

$\sin^2 A = 1/(1 + \cot^2 A)$

$\sin A = 1/\sqrt{(1 + \cot^2 A)}$

Finding sec A:

We know: $\tan A = 1/\cot A$

Also: $\sec^2 A = 1 + \tan^2 A$

$\sec^2 A = 1 + 1/\cot^2 A$

$\sec^2 A = (\cot^2 A + 1)/\cot^2 A$

$\sec A = \sqrt{(1 + \cot^2 A)}/\cot A$

Finding tan A:

$\tan A = 1/\cot A$

Answer:

$\sin A = 1/\sqrt{(1 + \cot^2 A)}$

$\sec A = \sqrt{(1 + \cot^2 A)}/\cot A$

$\tan A = 1/\cot A$

Example 12: If 3 cot A = 4, check whether $(1-\tan^2A)/(1+\tan^2A) = \cos^2A - \sin^2A$ or not.

Solution:

Given: 3 cot A = 4

Therefore: cot A = 4/3

This means: Base/Perpendicular = 4/3

So, Base = 4k, Perpendicular = 3k

Hypotenuse = $\sqrt{(16k^2 + 9k^2)} = 5k$

tan A = 3/4, sin A = 3/5, cos A = 4/5

LHS = $(1-\tan^2A)/(1+\tan^2A)$

$= [1 - (3/4)^2]/[1 + (3/4)^2]$

$= [1 - 9/16]/[1 + 9/16]$

$= [7/16]/[25/16]$

$= 7/25$

RHS = $\cos^2A - \sin^2A$

$= (4/5)^2 - (3/5)^2$

$= 16/25 - 9/25$

$= 7/25$

Since LHS = RHS = 7/25

Answer: YES, the equation is correct ✓

📝 PREVIOUS YEARS' BOARD QUESTIONS

2 MARK QUESTIONS

Q1. If $\sin A = 3/4$, calculate $\cos A$ and $\tan A$. (2020)

Solution:

Given: $\sin A = 3/4$

Perpendicular = 3k, Hypotenuse = 4k

Base = $\sqrt{(\text{Hypotenuse}^2 - \text{Perpendicular}^2)}$

Base = $\sqrt{(16k^2 - 9k^2)} = \sqrt{7k^2} = k\sqrt{7}$

$\cos A$ = Base/Hypotenuse = $k\sqrt{7}/4k$ = **$\sqrt{7}/4$**

$\tan A$ = Perpendicular/Base = $3k/(k\sqrt{7})$ = **$3/\sqrt{7}$ or $3\sqrt{7}/7$**

Answer: $\cos A = \sqrt{7}/4$, $\tan A = 3/\sqrt{7}$

Q2. Evaluate: $2 \tan^2 45° + \cos^2 30° - \sin^2 60°$ (2021)

Solution:

$= 2(1)^2 + (\sqrt{3}/2)^2 - (\sqrt{3}/2)^2$

$= 2 + 3/4 - 3/4$

$= \mathbf{2}$

Answer: 2

Q3. If 15 cot A = 8, find sin A and sec A. (2022)

Solution:

Given: 15 cot A = 8

cot A = 8/15

Base = 8k, Perpendicular = 15k

Hypotenuse = $\sqrt{64k^2 + 225k^2} = 17k$

sin A = 15k/17k = **15/17**

sec A = 17k/8k = **17/8**

Answer: sin A = 15/17, sec A = 17/8

3 MARK QUESTIONS

Q4. Prove that: (cos A/1-tan A) + (sin A/1-cot A) = sin A + cos A (2023)

Solution:

LHS = (cos A/1-tan A) + (sin A/1-cot A)

$= \cos A/(1 - \sin A/\cos A) + \sin A/(1 - \cos A/\sin A)$

$= \cos A/[(\cos A - \sin A)/\cos A] + \sin A/[(\sin A - \cos A)/\sin A]$

$= \cos^2 A/(\cos A - \sin A) + \sin^2 A/(\sin A - \cos A)$

$= \cos^2 A/(\cos A - \sin A) - \sin^2 A/(\cos A - \sin A)$

$= (\cos^2 A - \sin^2 A)/(\cos A - \sin A)$

$= (\cos A - \sin A)(\cos A + \sin A)/(\cos A - \sin A)$

$= \cos A + \sin A$

$= \sin A + \cos A$

= RHS ✓

Hence Proved

Q5. If tan (A+B) = √3 and tan (A-B) = 1/√3, 0° < A+B ≤ 90°, A > B, find A and B. (2024)

```
Solution:
Given: tan(A+B) = √3
From table: tan 60° = √3
Therefore: A + B = 60° ... (1)

Given: tan(A-B) = 1/√3
From table: tan 30° = 1/√3
Therefore: A - B = 30° ... (2)

Adding (1) and (2):
2A = 90°
A = 45°

From (1): 45° + B = 60°
B = 15°

Answer: A = 45°, B = 15°
```

5 MARK QUESTIONS

Q6. Prove that: $(\sin\theta - 2\sin^3\theta)/(2\cos^3\theta - \cos\theta) = \tan\theta$ (2022)

Solution:

LHS = $(\sin\theta - 2\sin^3\theta)/(2\cos^3\theta - \cos\theta)$

Taking $\sin\theta$ common in numerator and $\cos\theta$ in denominator:

$= \sin\theta(1 - 2\sin^2\theta)/[\cos\theta(2\cos^2\theta - 1)]$

Now, $1 - 2\sin^2\theta = 1 - \sin^2\theta - \sin^2\theta$

$= \cos^2\theta - \sin^2\theta$

$= \cos^2\theta - (1 - \cos^2\theta)$

$= 2\cos^2\theta - 1$

Therefore:

$= \sin\theta(2\cos^2\theta - 1)/[\cos\theta(2\cos^2\theta - 1)]$

$= \sin\theta/\cos\theta$

$= \tan\theta$

= RHS ✓

Hence Proved

⚠️ COMMON MISTAKES TO AVOID:

❌ **Mistake 1:** Confusing reciprocal and quotient relations

- Wrong: tan A = cos A/sin A
- Correct: tan A = sin A/cos A
- **Remember:** Tan = Top/Touch (Perpendicular/Base)

❌ **Mistake 2:** Not memorizing the standard angle table

- This table is CRUCIAL - you MUST memorize it!
- Practice writing it 20 times before exam

❌ **Mistake 3:** Identity confusion

- Wrong: $\sin^2 A + \cos^2 A = 2$
- Correct: $\sin^2 A + \cos^2 A = 1$
- Wrong: $1 + \tan^2 A = \text{cosec}^2 A$
- Correct: $1 + \tan^2 A = \sec^2 A$

❌ **Mistake 4:** Calculation with √

- Wrong: $1/\sqrt{2} = \sqrt{2}$
- Correct: $1/\sqrt{2} = \sqrt{2}/2$ (rationalize!)
- Wrong: $(\sqrt{3}/2)^2 = 3/2$
- Correct: $(\sqrt{3}/2)^2 = 3/4$

❌ **Mistake 5:** Forgetting to rationalize denominators

- Wrong: Final answer = $1/\sqrt{3}$
- Correct: Rationalize = $\sqrt{3}/3$
- Always rationalize unless question says otherwise

❌ **Mistake 6:** Using calculator values instead of exact values

- Wrong: sin 30° = 0.5
- Correct: sin 30° = 1/2 (keep in fraction form)

- Use exact values from table, not decimals!

❌ **Mistake 7:** Pythagoras theorem errors

- Wrong: Base = Hypotenuse + Perpendicular
- Correct: $\text{Hypotenuse}^2 = \text{Base}^2 + \text{Perpendicular}^2$

❌ **Mistake 8:** Not simplifying final answer

- Always simplify fractions: 4/8 = 1/2
- Always simplify radicals: $\sqrt{50} = 5\sqrt{2}$

💯 EXAM STRATEGY & TIME MANAGEMENT

Question Type	Marks	Time	Strategy
Finding one ratio from another	2	2-3 min	Draw triangle, use Pythagoras, find all sides, calculate ratio
Evaluating standard angles	2	2 min	Use memorized table values, simplify carefully
Simple identity proof	3	4-5 min	Take LHS or RHS, use identities, show all steps clearly
Angle finding problems	3	3-4 min	Match with table values, form equations, solve
Complex identity proof	5	7-8 min	Convert to sin/cos, use all three identities, simplify step-by-step
Application problems	3-5	5-7 min	Draw diagram, identify ratios, apply formulas, verify answer

⏰ Time Allocation Tips:

- **Read carefully:** Identify what's given and what to find (30 sec)
- **For identities:** Decide which side (LHS/RHS) is easier to simplify
- **Show ALL steps:** Even small steps get marks!
- **Use standard table:** Don't calculate sin 45° - use 1/√2 directly
- **Box final answer:** Make it clearly visible
- **Verify if time permits:** Substitute values to check identities
- **Practice speed:** Solve 50+ problems before exam

🎯 IMPORTANT FORMULAS - QUICK REFERENCE

1. SIX TRIGONOMETRIC RATIOS

sin A = P/H   cos A = B/H   tan A = P/B

cosec A = H/P   sec A = H/B   cot A = B/P

2. RECIPROCAL RELATIONS

sin A = 1/cosec A   cos A = 1/sec A   tan A = 1/cot A

cosec A = 1/sin A   sec A = 1/cos A   cot A = 1/tan A

3. QUOTIENT RELATIONS

tan A = sin A/cos A   cot A = cos A/sin A

4. THREE FUNDAMENTAL IDENTITIES

$$\sin^2 A + \cos^2 A = 1$$

$$1 + \tan^2 A = \sec^2 A$$

$$1 + \cot^2 A = \text{cosec}^2 A$$

5. DERIVED FORMS

$\sin^2 A = 1 - \cos^2 A$ $\cos^2 A = 1 - \sin^2 A$

$\tan^2 A = \sec^2 A - 1$ $\sec^2 A - \tan^2 A = 1$

$\cot^2 A = \text{cosec}^2 A - 1$ $\text{cosec}^2 A - \cot^2 A = 1$

📊 STANDARD ANGLES TABLE (MEMORIZE!)

Angle	0°	30°	45°	60°	90°
sin	0	1/2	1/√2	√3/2	1
cos	1	√3/2	1/√2	1/2	0
tan	0	1/√3	1	√3	∞
cosec	∞	2	√2	2/√3	1
sec	1	2/√3	√2	2	∞
cot	∞	√3	1	1/√3	0

🚀 LAST MINUTE REVISION CHECKLIST

☑ Theory to Remember:

- ☑ All 6 trigonometric ratios - definitions
- ☑ Standard angles table - write 20 times!
- ☑ Three fundamental identities - write 10 times!
- ☑ Reciprocal and quotient relations
- ☑ How to derive one ratio from another
- ☑ Pythagoras theorem application

☑ Quick Checks Before Writing Answer:

- ☑ Did I write the formula first?
- ☑ Are my standard angle values correct?
- ☑ Did I rationalize the denominator?
- ☑ Did I simplify square roots?
- ☑ Have I shown all steps?
- ☑ Is final answer boxed/highlighted?
- ☑ Did I verify using both sides (for identities)?

☑ Common Question Types:

- ☑ Find all ratios if one ratio is given
- ☑ Evaluate expressions using standard angles
- ☑ Prove trigonometric identities
- ☑ Find angles using table values
- ☑ Express one ratio in terms of another
- ☑ Solve right triangle problems
- ☑ Simplify trigonometric expressions

☑ Before Exam:

- ☑ Practice 100+ problems (minimum!)

- ☑ Solve last 5 years' board questions
- ☑ Time yourself - 2-8 min per question
- ☑ Memorize table - test yourself 10 times
- ☑ Revise all three identities and proofs
- ☑ Go through common mistakes list
- ☑ Practice identity proofs (20+ problems)
- ☑ Review all example problems

💡 EXPERT TIPS FOR SCORING FULL MARKS

How to Score 100% in Trigonometry:

- **1. Memorize perfectly:** Standard angles table must be at your fingertips
- **2. Formula first:** Always write the formula before substituting
- **3. Show ALL working:** Every single step matters for marks
- **4. Draw diagrams:** For word problems, draw right triangle
- **5. Rationalize always:** Unless specifically told not to
- **6. Simplify completely:** Don't leave $\sqrt{32}$, write $4\sqrt{2}$
- **7. For identities:** Show clear progression from one side to other
- **8. Use correct notation:** $\sin^2 A$ not $(\sin A)2$
- **9. Box final answer:** Make it stand out clearly
- **10. Verify if time permits:** Substitute values to check
- **11. Practice daily:** 30 minutes minimum for 30 days
- **12. Learn from mistakes:** Maintain error log

📖 PRACTICE QUESTIONS FOR SELF-ASSESSMENT

Section A: 2 Marks Questions

1. If $\sin A = 4/5$, find $\cos A$ and $\tan A$.
2. Evaluate: $\sin^2 30° + \cos^2 30°$
3. If $\tan \theta = 1/\sqrt{3}$, find the value of $\operatorname{cosec}^2\theta - \sec^2\theta$.
4. Evaluate: $(\cos 45°)/(\sec 30° + \operatorname{cosec} 30°)$
5. If $5 \tan \theta = 4$, find $(5 \sin \theta - 3 \cos \theta)/(5 \sin \theta + 2 \cos \theta)$

Section B: 3 Marks Questions

6. Prove that: $\sin^2 A/(1 - \cos A) = 1 + \cos A$
7. If $\sec \theta + \tan \theta = p$, show that $\sin \theta = (p^2 - 1)/(p^2 + 1)$
8. Evaluate: $(\tan 60° + 1)/(\tan 60° - 1)$
9. If $\sin(A + B) = 1$ and $\cos(A - B) = 1$, find A and B.
10. Express $\tan A$ in terms of $\sin A$.

Section C: 5 Marks Questions

11. Prove that: $\sqrt{[(1 + \sin A)/(1 - \sin A)]} = \sec A + \tan A$
12. Prove that: $(1 + \tan^2 A)/(1 + \cot^2 A) = (1 - \tan A)/(1 - \cot A)^2 = \tan^2 A$
13. Prove that: $(\sin A - \cos A + 1)/(\sin A + \cos A - 1) = 1/(\sec A - \tan A)$
14. If $\cos \theta + \sin \theta = \sqrt{2} \cos \theta$, show that $\cos \theta - \sin \theta = \sqrt{2} \sin \theta$
15. Prove that: $(\operatorname{cosec} A - \sin A)(\sec A - \cos A) = 1/(\tan A + \cot A)$

🎓 ADDITIONAL PRACTICE - MIXED QUESTIONS

Practice Q1: Prove that: $(1 + \cot A - \operatorname{cosec} A)(1 + \tan A + \sec A) = 2$

Hint: Expand and use identities $\sin^2 A + \cos^2 A = 1$

Practice Q2: If $\tan A + \sin A = m$ and $\tan A - \sin A = n$, show that $m^2 - n^2 = 4\sqrt{mn}$

Hint: Use $(a + b)^2 - (a - b)^2 = 4ab$

Practice Q3: In a right triangle ABC, right-angled at B, if $\tan A = 1/\sqrt{3}$, find the value of:
(i) $\sin A \cos C + \cos A \sin C$
(ii) $\cos A \cos C - \sin A \sin C$

Hint: First find angle A from table, then find angle C

Practice Q4: Prove that: $\sin^6 A + \cos^6 A = 1 - 3\sin^2 A \cos^2 A$

Hint: Use $a^3 + b^3 = (a + b)^3 - 3ab(a + b)$

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